MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

EEM2046 – ENGINEERING MATHEMATICS IV

(RE / TE)

23 OCTOBER 2019

9.00 a.m.– 11.00 a.m.

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 5 pages including the cover page.
2. Answer all questions.
3. The distribution of the marks for each question is given.
4. Please write all your answers in the answer booklet provided.
5. All necessary workings **MUST** be shown.

Question 1

(a) Given a linear programming model as below:

$$\begin{aligned}
\text{Maximize} \quad & 12x_1 + 6x_2 + 4x_3 \\
\text{Subject to :} \quad & 4x_1 + 2x_2 + x_3 \leq 60 \\
& 2x_1 + 3x_2 + 3x_3 \leq 50 \\
& x_1 + 3x_2 + x_3 \leq 45 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

By introducing slack variables into the above model, rewrite the model into standard linear programming form. Then solve the problem using simplex algorithm and indicate the optimal values.

[19 marks]

(b) Construct the dual problem for the following primal problem:

$$\begin{aligned}
\text{Maximize :} \quad & z = 6x_1 + 5x_2 \\
\text{Subject to :} \quad & 3x_1 + 3x_2 \leq 18 \\
& 12x_1 + 8x_2 \leq 48 \\
& 8x_1 + 2x_2 \leq 8 \\
& x_1, x_2 \geq 0
\end{aligned}$$

[6 marks]

Continued...

Question 2

a) Suppose that X and Y have joint probability density function

$$f(x,y)=\begin{cases} Cy^2e^{-x} & x>1, 0<y<2 \\ 0 & otherwise \end{cases}$$

where C is a constant to be sought.

i) Determine the value of C. Leave your answer in natural logarithm e. [7 marks]

ii) Find the marginal probability density function of X and Y, respectively. [8 marks]

iii) Determine whether X and Y are independent. [4 marks]

b) Given that probability density function(pdf) of a random variable X is

$$f_X(x)=\begin{cases} e^{-\alpha x} & x>0 \\ 0 & otherwise \end{cases}$$

Find the pdf for $Y=5X$, where α is a constant. [6 marks]

Continued...

Question 3

Consider a Markov chain with state space $\{0,1,2,3\}$, and the transition probability matrix as follows:

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{5} & \frac{4}{5} & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

i) Draw the state transition diagram [3 marks]

ii) Specify the recurrent or transient states. [4 marks]

iii) Find the period for each of the recurrent states. [2 marks]

iii) Initially, the particle is in position 2. What is the probability that the particle will be in position 1 after 2 transitions ? [5 marks]

iv) If the process starts from state 3, then find $f_{31}^{(3)}$, the probability to jump to state 1 at the third transition for the first time, using the formula

$$f_{ij}^{(m)} = P_{ij}^{(m)} - \sum_{l=1}^{m-1} f_{ij}^{(l)} P_{jj}^{(m-l)}$$ [11 marks]

Continued...

Question 4

a) Evaluate the integral $\oint_C \frac{dz}{z(z+2)}$, where C is the any rectangle containing the points $z = 0$and $z = -2$inside it. [9 marks]

b) Evaluate the integral $\int_C |z|^2 dz$, where C is a line from $z_1 = 1$ to $z_2 = -1 - i$.

[7 marks]

c) Find an analytic function $f(z)$ by finding the conjugate of the harmonic function

$u(x, y) = x^3 - 3xy^2 + y$ [9 marks]

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